General Certificate of Education
Advanced Subsidiary (AS) and Advanced Level

MATHEMATICS **S2**

Probability & Statistics 2

Additional materials:
Answer paper
Graph paper
List of Formulae

SPECIMEN PAPER

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 60.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 3 printed pages and 1 blank page.

1 Before being packed in boxes, apples in a fruit-packing plant have to be checked for bruising. The apples pass along a conveyor belt, and an inspector removes any of the apples that are badly bruised. Badly bruised apples arrive at random times, but at a constant average rate of 1.8 per minute.

(i) Find the probability that at least one badly bruised apple arrives in a one-minute period. [3]

(ii) In a period of a minutes, the probability of at least one badly bruised apple arriving is 0.995. Find the value of a. [3]

2 A student answers a test consisting of 16 multiple-choice questions, in each of which the correct response has to be selected from the four possible answers given. The student only gets 2 of the questions correct, and the teacher remarks that this 'shows that the student did worse than anyone would do just by guessing the answers'. The probability of the student answering a question correctly is denoted by p, assumed to be the same for all the questions in the test.

(i) State suitable null and alternative hypotheses, in terms of p, for a test to examine whether the teacher's remark is justified. [2]

(ii) Carry out the test, using a 10% significance level, and state your conclusion clearly. [4]

3 Lessons in a school are supposed to last for 40 minutes. However, a Mathematics teacher finds that pupils are usually late in arriving for his lessons, and that the actual length of teaching time available can be modelled by a normal distribution with mean 34.8 minutes and standard deviation 1.6 minutes.

(i) Find the probability that the length of teaching time available will be less than 37.0 minutes. [2]

(ii) The probability that the length of teaching time available exceeds m minutes is 0.75. Find m. [3]

The teacher has a weekly allocation of 5 lessons with a particular class. Assuming that these 5 lessons can be regarded as a random sample, find the probability that the mean length of teaching time available in these 5 lessons will lie between 34.0 and 36.0 minutes. [4]

4 It is given that 93% of children in the UK have been immunised against whooping cough. The number of children in a random sample of 60 UK children who have been immunised is X, and the number who have not been immunised is Y. State, with reasons, which of X or Y has a distribution which can be approximated by a Poisson distribution. [3]

Using a Poisson approximation, find the probability that at least 58 children in the sample of 60 have been immunised against whooping cough. [3]

Three random samples, each of 60 UK children, are taken. Find the probability that in one of these samples exactly 59 children have been immunised while in each of the other two samples exactly 58 children have been immunised. Give your answer correct to 1 significant figure. [3]

5 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} kx^2(3-x) & 0 \le x \le 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{4}{27}$, and find $\mathrm{E}(X)$. **[5]**

(ii) Find $\mathrm{P}(X < 2)$. **[2]**

(iii) Use your answer to part **(ii)** to state, with a reason, whether the median of X is less than 2, equal to 2 or greater than 2. **[2]**

6 The 'reading age' of children about to start secondary school is a measure of how good they are at reading and understanding printed text. A child's reading age, measured in years, is denoted by the random variable X. The distribution of X is assumed to be $\mathrm{N}(\mu, \sigma^2)$. The reading ages of a random sample of 80 children were measured, and the data obtained is summarised by $\Sigma x = 892.7$, $\Sigma x^2 = 10\,266.82$.

(i) Calculate unbiased estimates of μ and σ^2, giving your answers correct to 2 decimal places. **[3]**

(ii) Previous research has suggested that the value of μ was 10.75. Determine whether the evidence of this sample indicates that the value of μ is now different from 10.75. Use a 10% significance level for your test. **[5]**

(iii) State, giving a brief reason, whether your conclusion in part **(ii)** would remain valid if

(a) the distribution of X could not be assumed to be normal, **[1]**

(b) the 80 children were all chosen from those starting at one particular secondary school. **[1]**

7 The breaking strength of a certain type of fishing line has a normal distribution with standard deviation 0.24 kN. A random sample of 10 lines is tested. The mean breaking strengths of the sample and of the population are $\bar{x}$ kN and μ kN respectively. The null hypothesis $\mu = 8.75$ is tested against the alternative hypothesis $\mu < 8.75$ at the $2\frac{1}{2}\%$ significance level.

(i) Show that the range of values of $\bar{x}$ for which the null hypothesis is rejected is given by $\bar{x} < 8.60$, correct to 2 decimal places. **[4]**

(ii) Explain briefly what is meant, in the context of this question, by a Type I error, and state the probability of making a Type I error. **[2]**

(iii) Explain briefly what is meant, in the context of this question, by a Type II error, and find the probability of making a Type II error when $\mu = 8.50$. **[5]**

1	(i) $P(\text{at least one}) = 1 - e^{-1.8}$ $= 0.835$	B1 M1 A1 3	For any use of Po(1.8) For use of complementary probability
	(ii) $1 - e^{-1.8a} = 0.995$ $-1.8a = \ln 0.005$ $a = 2.94$	B1 M1 A1 3	For correct use of logs in solving for a
2	(i) Null hypothesis: $p = \frac{1}{4}$ Alternative hypothesis: $p < \frac{1}{4}$	B1 B1 2	
	(ii) Under the NH, number correct $\sim B(16, \frac{1}{4})$ Hence P(2 or fewer correct) = 0.1971 This is greater than 0.1 (not significant) There is insufficient evidence to justify the teacher's suggestion that the score was worse than would be produced by pure guesswork	M1 A1 M1 A1✓ 4	May be implied Using tables or direct calculation For comparing with the significance level
3	(i) $P(T < 37.0) = \Phi\left(\frac{37.0 - 34.8}{1.6}\right) = \Phi(1.375) = 0.915$	M1 A1 2	Standardising and using tables Correct answer 0.915
	(ii) $\frac{m - 34.8}{1.6} = -0.674$ Hence $m = 33.7$	M1 B1 A1 3	Equating standardised m to a z value For use of $(\pm)0.674$ in an equation
	$P(34.0 < \bar{T} < 36.0) = \Phi\left(\frac{36.0 - 34.8}{1.6/\sqrt{5}}\right) - \Phi\left(\frac{34.0 - 34.8}{1.6/\sqrt{5}}\right)$ $= \Phi(1.677) - \{1 - \Phi(1.118)\}$ $= 0.821$	M1 A1 M1 A1 4	For using $N(34.8, 1.6^2/5)$ For both end-points standardised correctly Correct process for prob *between* end-points
4	$X \sim B(60, 0.93)$, $Y \sim B(60, 0.07)$ Hence Y is suitable for a Poisson approximation, since n is large and p is small	M1 A1 A1 3	For either binomial distribution identified For correct conclusion For correct justification
	$Y \sim Po(4.2)$ Required probability is $P(Y \le 2)$ i.e. 0.210	B1 M1 A1 3	May be implied For attempted evaluation of relevant Po prob Using tables or direct calculation
	Required probability is $4.2\,e^{-4.2} \times \left(\frac{4.2^2}{2} e^{-4.2}\right)^2 \times 3$ i.e. 0.003 to 1 sf	B1✓ B1 B1✓ 3	For $p_1 \times p_2 \times p_2$, using Poisson or binomial For correct factor of 3 Follow through wrong value of 4.2 only

5	(i) $k\int_0^3 (3x^2 - x^3)\,dx = 1$	M1		Equating to 1 and attempting to integrate
	$k\left[x^3 - \frac{1}{4}x^4\right]_0^3 = 1$	A1		For correct integration
	$k\left(27 - \frac{81}{4}\right) = 1 \Rightarrow k = \frac{4}{27}$	A1		Given answer correctly obtained
	$E(X) = \frac{4}{27}\int_0^3 (3x^3 - x^4)\,dx$	M1		For correct application of $\int x\,f(x)\,dx$
	$= \frac{9}{5}$	A1	5	
	(ii) $P(X<2) = \frac{4}{27}\left[x^3 - \frac{1}{4}x^4\right]_0^2$	M1		
	$= \frac{16}{27}$	A1	2	Or equivalent decimal answer
	(iii) $\frac{16}{27}$ is greater than $\frac{1}{2}$	M1		For comparison of answer (ii) with $\frac{1}{2}$
	Hence the median is less than 2	A1√	2	
6	(i) $\bar{x} = \frac{892.7}{80} = 11.16$ to 2dp	B1		For correct value 11.16
	$s^2 = \frac{1}{79}\left(10\,266.82 - \frac{892.7^2}{80}\right)$	M1		For this expression, or equivalent
	$= 3.87$ to 2dp	A1	3	
	(ii) $H_0 : \mu = 10.75$, $H_1 : \mu \neq 10.75$	B1		Both hypotheses
	Test statistic is $z = \frac{11.16 - 10.75}{\sqrt{3.87/80}} = 1.86$	M1		Standardising attempt using $\sqrt{s^2/80}$
		A1√		Correct value; follow their s
	This is greater than critical (2 tail) value $z = 1.645$	M1		Or comparing $\Phi(1.86)$ with 5%
	There is evidence to suggest that the value of μ is now different	A1√	5	
	(iii) (a) Still valid, since the sample size (80) is large enough to appeal to the CLT	B1	1	Allow any reasoned conclusion mentioning the CLT
	(b) Not valid, since the children starting at one school may not be representative of all children of this age.	B1	1	For conclusion and reason
7	(i) Critical value is $8.75 - 1.96 \times \frac{0.24}{\sqrt{10}}$	M1		Calculation of correct form $8.75 - z \times$ S.E.
		B1		Relevant use of -1.96
		A1		Relevant use of $0.24/\sqrt{10}$
	i.e. reject null hypothesis when $\bar{x} < 8.60$	A1	4	
	(ii) NH $\mu = 8.75$ would be rejected when the mean breaking strength is in fact 8.75 kN	B1		
	P(Type I error) = 0.025	B1	2	
	(iii) NH $\mu = 8.75$ would be accepted when the mean breaking strength is in fact less than 8.75 kN	B1		
	Type II error occurs when $\bar{x} > 8.60$	B1		May be implied
	Probability is $1 - \Phi\left(\frac{8.60 - 8.50}{0.24/\sqrt{10}}\right)$	M1		Using normal distribution with mean 8.50
		A1		Correct standardising, and use of tables
	$= 0.0938$	A1	5	